Hans Wehrli, 12.9.2013

A Topos Foundation for Theories of Physics

I. *Comments to* Andreas Döring and C.J. Isham (2007): I. – IV. A Topos Foundation for Theories of Physics

1. Döring and Isham (a specialist for quantumgravitation) develop a language, logic and axiomatic for physical theories. The goal is not to describe the known physical theories by just another language but to open the way for a theory that combines relativity, quantum theory and GUT and possibly can even formulate a theory of the universe (ToE). The authors are sceptical to accept some apriori suppositions such as spatio-temporal concepts, continuum, real and complex numbers, Copenhagen interpretation of QT or Boolean logic (principle of excluded middle). But their careful effort does not succeed.

2. None the less their paper is a good basis for the development of a new physical language based on topos (category) theory or even of a ToE. But for this purpose one should dispense with the usual understanding of some notions that contradict the metaphysics of empirical perception such as the interpretation of reality or state. Instead one must add a few often neglected notions of metaphysics such as subject, information(flow), order, chirality and measurement. The main new notion is the event as the only entity which can be empirically perceived and therefore is real.

3. The authors are aware of the main problem of physical theories. The theories are expected to describe things that principally cannot be precisely known, perceived or measured. Any theory that describes the universe as a whole cannot at the same time describe all details of all small objects. And any theory which describes elementary particles cannot describe any possible hidden parameters i.e. global influences of the rest of the universe on these particles (Kochen-Specker theorem). Therefore there is an inherent uncertainty in every physical theory, be it that the language which describes the universe neglects many details, be it that the language which describes any small objects neglects the influence of the universe.

4. The authors try to solve the problem by introducing separate languages for each physical context and defining a system that translates all such languages into each other by topos theory. But not all the problems can be solved this way, the new system of languages is rather complicated and the authors cannot give a single explicit example of a physical theory that is actually improved by their approach.

5. The language describing the physical system S contains only two ground symbols, namely the state object Σ and the quantity value object $\mathcal{R}$ which is related to the set of real numbers $\mathbb{R}$ (which forms an abelian ring) and which is dependent on the theory type Φ. Σ as a state is generally something that is supposed to exist in time. But the only real entity that exists is the event: Something remains constant wheras something else changes. Not a state is observed but the change of the state. The states themselves are not real; they are part of the mathematical model or theory used. Instead of states a set of events should be the ground symbol of the new language. The suggested introduction of an extra symbol 'M' in the language for space-time is not required.

6. The authors describe the states by Hilbert spaces $\mathcal{H}$. Since events in many cases are not local it is not clear whether they can as well be described by Hilbert spaces. The mathematical model must be able to describe non local virtual particles or events as well as local and real ones.

7. Since every measurement is a counting of events, not $\mathbb{R}$ should be used for the quantity values but rather the natural or rational numbers $\mathbb{N}$ or $\mathbb{Q}$.

8. There is no observation and no counting without a sort of order and no order without chirality (in an abstract sense). Therefore an axiom of chirality must be introduced into the new theory.

9. It makes sense to introduce truth values *v* and truth objects. Although the measured values of the past are known and true, this is not the case for the future because the measured real values are changed by the event of measurement. In addition there might be unknown influences by the rest of the universe which change the measured value. A similar argument leads to an uncertainty with cosmological observations where the internal properties of most subobjects are neglected.

10. The authors mention vaguely the idea that time could later be introduced in their theory and that it might have a new, more relative meaning. Time should be a consequence of order and chirality. These are the sources of causality, and causality connects events of the past with events of the future. Time does not have to be absolute; there might be different times of the observing subject and the observed subsystem, and also time directions might be relative e.g. with antiparticles. Even whether something is interpreted as space or rather as time might be relative, e.g. within balck holes. It seems that the language system of the authors allowes all these ideas.

11. The authors suggest that a linguistic analogue of the natural numbers could be introduced. Their language could then be augmented with the axioms of Peano arithmetic – an interesting idea. Other axioms are Abelian, basic language axioms and natural number axioms.

12. The authors introduce a "daseinisation" procedure δ by means of truth values and topos theory. It is supposed to serve as a bridge between the intrinsic instrumentalism of standard quantum formalism and a full 'neo-realism' in which the truth value associated with the language is considered to be real. The advantage of such a new philosophical interpretation of reality is not made clear. It seems that the usual replacement of Boolean by Heynting (or quantum) logic in which the principle of the excluded middle (tertium non datur) does not hold serves the same goal.

13. The authors think that their language allowes tools for constructing theories that go beyond quantum theory and do not use Hilbert spaces, path integrals, continuum of real or complex numbers to play a fundamental role. One simple, but mathematically rich example arises from the theory of *M*-sets. Here *M* is a monoid and, like all monoids, can be viewed as a category, a monoid is 'complementary' to a partially-ordered set. In a monoid there is only one object, but plenty of arrows from that object to itself; whereas in a partially-ordered set there are plenty of objects, but at most one arrow between any pair of objects. Thus a partially-ordered set is the most economical category with which to capture the concept of 'contextual logic'. On the other hand, the logic associated with a monoid is non-contextual as there is only one object in the category (conclusion on p. 34f

of paper III). That is exactly the way my theory works when it describes all elementary particles.

14. The authors try to formulate a single 'category of systems' that combines several different, possibly disjoint categories of physical interest. They admit that actually there is only one true 'system', and that is the universe as a whole. Concomitantly, there is just one local language, and one topos. Since in my theory every black hole is a new universe and elementary particles are treated as mini-black holes, it should be possible to describe these by distinct true categories without any hidden parameters. These should appear not before the interactions between those particles are described. Therefore one should start a new theory beginning with the description of the simplest elementary particle, i.e. the neutrino, followed by the other leptons and then the quarks, as I proceeded with the development of my theory. Then the next step would be the description of the interaction between neutrinos, where gravity und the notion of distance are introduced, followed by the interaction between interactions that lead to the relativity theories and to the phenomenon of 'entanglement'. When the possibility of black holes within black holes with their specific action to the outside is added to the theory, the theory porobably is complete, i.e. it becomes a theory of everything ToE or – as the authors call it – 'un gros topos'.

II. Formal Language of Physics and Principal Steps for its Application to my Theory

1. The following ideas for a language to describe physics are based on the paper of A.Döring and C.J.Isham: A Topos Foundation for Theories of Physics: I.-IV. (2007). Since I am not a mathematician the ideas must still be translated into a correct formal language. The purpose of my ideas is to allow a mathematical formulation of my theory described in H.Wehrli: Metaphysics. Chirality as the Basic Principle of Physics (2008).

2. The **language $\mathcal{L}(S)$** describes a physical **system S**. "Physical" means that the system is principally observable by empirical perception. The system S as a whole is the **universe**.

3. The **set of events $\mathcal{C}_{\Phi,S} \rightarrow \mathcal{R}_{\Phi,S}$ of theory type Φ** is represented by a quantity value object $\mathcal{R}_{\Phi,S}$.

4. **$\tau_{\Phi}(S)$ is the topos** applied to S.

5. S consists of subsets called objects. The **basic object P** is the smallest possible object. It is one, has no parts, cannot be devided, is identical with itself, has no internal properties and corresponds to a point in topology. The basic object exists but it cannot be empirically observed; so it is not real. S consists of $\{P_n\}$, where $n = 1, 2, 3, \ldots n$.

6. All P in S are related to each other. The **relation r** is expressed by $r = (P_n , P_m)$. In $\{P_n\}$ there are n $(n -1)$ relations r, without considering any relations of relations and so on.

7. **Axiom of individuality**: $P_n \neq P_m$, i.e. two different P can never have the same relations to all other P.

8. **Axiom of order** (or chirality): $(P_n , P_m) \neq (P_m , P_n)$.

9. The duality of every r is called its **orientation**.

10. **Axioms of numbers**: Since every physical measurement is a counting of events, the natural numbers ($\mathbb{N}$) and rational numbers ($\mathbb{Q}$) are introduced by the Peano axioms, but without any irrational, complex or infinite numbers.

11. The **dimensionality *d*** is the number of independent r_i that can be produced by multiplying r by $\mathbb{N}$. $d(S_i) \leq i - 1$.

12. The **between relation** $\boldsymbol{r}_{\leftrightarrow}$ means that $P_{\leftrightarrow}$ reduces the dimensionality of S by 1. P is said to be in a between state.

13. An **event** occurs, if any P_m changes its relation to $P_{1, 2, \ldots i}$ by getting in a between relation to any of these other P.

14. A **process** is a change of the state of P by a function $f_{\leftrightarrow}$. There are 4 such possible states:

    $P_{\leftrightarrow}$ is in the between state and is changed by $f_{\leftrightarrow}$ into the away state $P_{\mapsto}$, i.e. into a state that does not lead to an event.
    $P_{\mapsto}$ is in the away state, which by $f_{\leftrightarrow}$ is transformed into the return state $P_{\leftarrow}$, i.e. a state that neither leads to an event nor not to an event.
    $P_{\leftarrow}$ is in the return state, which by $f_{\leftrightarrow}$ is transformed into the approach state $P_{\mapsto\leftarrow}$, i.e. a state that leads to an event.
    $P_{\mapsto\leftarrow}$ is in the approach state, which by $f_{\leftrightarrow}$ is transformed into a between state $P_{\leftrightarrow}$, i.e. a state that produces an event.

    These states are not internal properties of P but different relations to any other set of 3 P of the universe.

15. f is a morphism $\Rightarrow$, not an isomorphism $\Leftrightarrow$. From this function the **arrow of time** and the **chirality of space** can be derived.

16. A **particle** is a (not basic) object $S_{Pi, \ldots}$ with perpetual internal processes that do not change S and its orientation.

17. Particles as processes in classical space are only possible with **3-dimensionality**. Proof: …

18. **Information** = number of independent orientations in S. From this definition energy, chaos, black hole and thermodynamics can be derived. This definition should allow the prediction of the three neutrino masses or at least of the ratio of the three masses, which up to now could not be measured. Experiments only show that the masses are very small but not zero, and that probably the difference between the three masses is small.

19. The **distance** between two subsystems S_m and S_n is a rational number $\mathbb{Q}_{m,n}$ of events that would happen in a process where a P is tranferred from S_m to S_n. To evaluate $\mathbb{Q}_{m,n}$ probably "secondary events" must be taken into consideration. These should be defined by secondary between-relations within the whole universe, probably with relations of relations and so on.

20. **Physical measurement** is a counting of events which occur when the state of the observed system S changes. Such changes are always changes of an orientation within S. The information content of the observation can be expressed by the number of events transferred from the changing object to the subject, i.e. by the frequency of the process. The frequency is the result of a comparison of the number of observed events with the number of events within the observing subject. It corresponds to the change of energy during the process of the change of the state of the object.